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# Constraints from Neutrino Oscillation Experiments on the Effective Majorana Mass in Neutrinoless Double $\beta$ -Decay

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## Abstract

We determine the possible values of the effective Majorana neutrino mass  $|\langle m \rangle| = |\sum_j U_{ej}^2 m_j|$  in the different phenomenologically viable three and

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four-neutrino scenarios. The quantities  $U_{\alpha j}$  ( $\alpha = e, \mu, \tau, \dots$ ) denote the elements of the neutrino mixing matrix and the Majorana neutrino masses  $m_j$  ( $j = 1, 2, 3, \dots$ ) are ordered as  $m_1 < m_2 < \dots$ . Assuming  $m_1 \ll m_3$  in the three-neutrino case and  $m_1 \ll m_4$  in the four-neutrino case, we discuss, in particular, how constraints on  $|\langle m \rangle|$  depend on the mixing angle relevant in solar neutrino oscillations and on the three mass-squared differences obtained from the analyses of the solar, atmospheric and LSND data. If neutrinoless double  $\beta$ -decay proceeds via the mechanism involving  $|\langle m \rangle|$ , conclusions about neutrinoless double  $\beta$ -decay can be drawn. If one of the two viable four-neutrino schemes (Scheme A) is realized in nature,  $|\langle m \rangle|$  can be as large as 1 eV and neutrinoless double  $\beta$ -decay could possibly be discovered in the near future. In this case a Majorana CP phase of the mixing matrix  $U$  could be determined. In the other four-neutrino scheme (Scheme B) there is an upper bound on  $|\langle m \rangle|$  of the order of  $10^{-2}$  eV. In the case of three-neutrino mixing the same is true if the neutrino mass spectrum is hierarchical, however, if there exist two quasi-degenerate neutrinos and the first neutrino has a much smaller mass, values of  $|\langle m \rangle|$  as large as  $\sim 0.1$  eV are possible.

14.60.Pq, 14.60.St

## I. INTRODUCTION

The observation of a significant up-down asymmetry of atmospheric multi-GeV muon events by the Super-Kamiokande collaboration [1] is considered as a strong evidence in favour of neutrino masses and mixing. The further investigation of neutrino properties and the understanding of the origin of neutrino masses and mixing is a very important issue of present day physics. One of the most fundamental problems of the physics of neutrinos is the question of the nature of massive neutrinos: Are massive neutrinos Dirac particles possessing some conserved lepton number or truly neutral Majorana particles having all lepton numbers equal to zero? Neutrino oscillation experiments cannot answer this question because the additional phases of the neutrino mixing matrix in the Majorana case do not enter into the transition probabilities in vacuum [2] as well as in matter [3].

A direct way to reveal the nature of massive neutrinos is to investigate processes in which the total lepton number is not conserved. The most promising process of this type is neutrinoless double  $\beta$ -decay of nuclei (for reviews see [4]),

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^- . \quad (1.1)$$

In the framework of the standard left-handed weak interactions with the Hamiltonian for  $\beta$ -decay given by

$$\mathcal{H}_\beta = \sqrt{2}G_F \bar{e}_L \gamma_\alpha \nu_{eL} j^\alpha + \text{h.c.} , \quad (1.2)$$

where  $j^\alpha$  is the weak hadronic current, and with the mixing of Majorana neutrinos

$$\nu_{\alpha L} = \sum_j U_{\alpha j} \nu_{jL} \quad \text{with} \quad \alpha = e, \mu, \tau, \dots , \quad (1.3)$$

where  $\nu_j = \nu_j^c \equiv C\bar{\nu}_j^T$  ( $j = 1, 2, 3, \dots$ ) is the field of a Majorana neutrino with mass  $m_j$ , the matrix element of the process (1.1) is proportional to the effective Majorana neutrino mass

$$|\langle m \rangle| = \left| \sum_j U_{ej}^2 m_j \right| , \quad (1.4)$$

which originates from the neutrino propagator

$$\langle 0 | T [\nu_{eL}(x_1) \nu_{eL}^T(x_2)] | 0 \rangle . \quad (1.5)$$

In this paper, assuming that massive neutrinos are Majorana particles, we will present constraints on the parameter  $|\langle m \rangle|$  that can be obtained from results of neutrino oscillation experiments [5–15,17] in the framework of the two possible schemes of mixing of four massive neutrinos that can accommodate the results of all existing neutrino oscillation experiments (Schemes A and B [16]) and two schemes of mixing of three massive neutrinos with  $m_1 \ll m_3$  that can accommodate the results of all experiments except the results of the LSND experiment (the scheme with a hierarchy of neutrino masses and the scheme with the reversed hierarchy of neutrino masses, i.e., with quasi-degenerate masses  $m_2, m_3$ ) It will be shown that in one of the four-neutrino mixing schemes (Scheme A) the effective Majorana mass

$|\langle m \rangle|$  can be as large as the “LSND mass”  $\sqrt{\Delta m_{\text{LSND}}^2}$  lying between 0.4 and 1.4 eV. In the three-neutrino mixing scheme with reversed mass hierarchy  $|\langle m \rangle|$  can be equal to the “atmospheric neutrino mass”  $\sqrt{\Delta m_{\text{atm}}^2}$  ranging from 0.03 to 0.1 eV. In two other schemes the effective Majorana mass is strongly suppressed with respect to the masses of the heaviest neutrinos. We will show that in the case of the three-neutrino mass hierarchy and in Scheme B of the mixing of four neutrinos the upper bound on  $|\langle m \rangle|$  is in the range of a few  $10^{-2}$  eV.

Many experiments on the search for  $(\beta\beta)_{0\nu}$  decay of different nuclei are going on at present (for a review see Ref. [18]). Up to now neutrinoless double  $\beta$ -decay has not been found, therefore, only lower bounds on the half-life of the  $(\beta\beta)_{0\nu}$  decay modes can be inferred from the experimental data. The most stringent limit was obtained in the  $^{76}\text{Ge}$  Heidelberg – Moscow experiment [19]. The latest result of this experiment is

$$T_{1/2} > 5.7 \times 10^{25} \text{ y}. \quad (1.6)$$

Concerning other nuclei, the best limit for  $^{136}\text{Xe}$  ( $T_{1/2} > 4.4 \times 10^{23} \text{ y}$ ) has been obtained by the Caltech – PSI – Neuchâtel Coll. [20] and for  $^{130}\text{Te}$  ( $T_{1/2} > 7.7 \times 10^{22} \text{ y}$ ) by the Milano Group [21]. For a list of recent experimental results see Table 2 in Ref. [18].

From the lower bounds on  $T_{1/2}$  upper bounds on the effective Majorana mass  $|\langle m \rangle|$  can be inferred. Taking the result of the  $^{76}\text{Ge}$  experiment, it was found (see references cited in [19]) that

$$|\langle m \rangle| \lesssim 0.2 - 0.6 \text{ eV}. \quad (1.7)$$

Such bounds are obtained by using the results of the calculations of the nuclear matrix elements of the relevant isotopes, performed by different groups in the framework of the Shell Model or the Quasiparticle Random Phase Approximation. For a discussion of calculations of  $(\beta\beta)_{0\nu}$  nuclear matrix elements see Ref. [18].

Running experiments or experiments under preparation plan to reach a sensitivity of  $|\langle m \rangle| \sim 10^{-1}$  eV. The feasibility of a new generation of  $(\beta\beta)_{0\nu}$  decay experiments, CUORE [21] and GENIUS [22], which could reach the region  $|\langle m \rangle| \sim 10^{-2}$  eV, is under consideration at the moment.

The mechanism for  $(\beta\beta)_{0\nu}$  decay based on the propagator (1.5) is not unique. In addition to the Majorana neutrino exchange, there can be other possible mechanisms involving physics beyond the standard model: right-handed currents, SUSY with violation of R-parity, mechanisms with scalars, etc. (for a review see Ref. [23]). It has been argued that, independently of the underlying mechanism, an observation of  $(\beta\beta)_{0\nu}$  decay would be an evidence for a non-zero *Majorana* neutrino mass. In Ref. [24] the argument is phrased in terms of Feynman graphs, whereas in Ref. [25] symmetry reasons are given. However, these arguments do not allow to calculate the Majorana neutrino mass. Actually, the contribution to the Majorana neutrino mass suggested by these arguments is exceedingly small. Thus, it is clear that any information on  $|\langle m \rangle|$  that can be obtained from neutrino oscillation data could help to reveal the true mechanism of  $(\beta\beta)_{0\nu}$  decay.

## II. NEUTRINOLESS DOUBLE $\beta$ -DECAY IN FOUR-NEUTRINO SCHEMES

Results from many neutrino oscillation experiments are available at present. Convincing evidence in favour of neutrino masses and oscillations have been obtained in atmospheric

[1,26] and solar neutrino experiments [27,28]. Observation of  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  oscillations has been claimed by the LSND collaboration [29]. On the other hand, in different reactor and accelerator short-baseline (SBL) experiments (see [30]) and in the long-baseline (LBL) reactor experiment CHOOZ [31] no indications in favour of neutrino oscillations were found.

From the analysis of all these data it follows that there are three different scales of neutrino mass-squared differences  $\Delta m^2$ :

$$\Delta m_{\text{solar}}^2 \sim 10^{-5} \text{ eV}^2 \text{ or } 10^{-10} \text{ eV}^2, \quad \Delta m_{\text{atm}}^2 \sim 10^{-3} \text{ eV}^2, \quad \Delta m_{\text{LSND}}^2 \sim 1 \text{ eV}^2. \quad (2.1)$$

In order to accommodate all these data we need to assume that at least four massive neutrinos exist in nature. This means that, in addition to the three flavour neutrinos, sterile neutrinos must exist.

In the framework of the minimal scheme with four massive neutrinos [32–34,7,16], it was shown in Ref. [16] that from the six possible types of mass spectra with mass-squared differences (2.1) only two are compatible with all data, including the latest Super-Kamiokande data on the measurement of the up-down asymmetry of multi-GeV atmospheric muon events:

$$(A) \quad \underbrace{\overbrace{m_1 < m_2}^{\text{atm}} \ll \overbrace{m_3 < m_4}^{\text{solar}}}_{\text{LSND}} \quad \text{and} \quad (B) \quad \underbrace{\overbrace{m_1 < m_2}^{\text{solar}} \ll \overbrace{m_3 < m_4}^{\text{atm}}}_{\text{LSND}}. \quad (2.2)$$

In order to determine the effective Majorana mass  $|\langle m \rangle|$  (1.4) we need to know the neutrino masses and the elements  $U_{ej}$  of the neutrino mixing matrix. Information on these elements can be obtained from the results of reactor and solar neutrino experiments. In the framework of Schemes A and B the  $\bar{\nu}_e$  survival probability in SBL experiments is given by [16]

$$P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}^{\text{SBL}} = 1 - \frac{1}{2} B_{e;e} \left( 1 - \cos \frac{\Delta m_{41}^2 L}{2p} \right), \quad (2.3)$$

where  $\Delta m_{41}^2 = m_4^2 - m_1^2$  is the largest neutrino mass-squared difference,  $L$  is the source – detector distance,  $p$  denotes the neutrino momentum and

$$B_{e;e} = 4 \sum_{j=3,4} |U_{ej}|^2 \left( 1 - \sum_{j=3,4} |U_{ej}|^2 \right) \quad (2.4)$$

represents the oscillation amplitude. Taking into account the results of the solar neutrino experiments, the negative results of the reactor experiments allow to deduce the bounds

$$\sum_{j=3,4} |U_{ej}|^2 \geq 1 - a_e^0 \quad \text{in Scheme A} \quad (2.5)$$

and

$$\sum_{j=3,4} |U_{ej}|^2 \leq a_e^0 \quad \text{in Scheme B.} \quad (2.6)$$

The quantity  $a_e^0$  is defined by

$$a_e^0 = \frac{1}{2} \left( 1 - \sqrt{1 - B_{e;e}^0} \right), \quad (2.7)$$

where  $B_{e;e}^0$  is the upper bound on the amplitude  $B_{e;e}$ . For each value of the mass-squared difference  $\Delta m_{41}^2$ , the value of  $B_{e;e}^0$  is obtained from the exclusion curves in the plane of oscillation parameters obtained in  $\bar{\nu}_e$  disappearance experiments. Consequently,  $B_{e;e}^0$  and  $a_e^0$  are functions of  $\Delta m_{41}^2$ . Using the result of the Bugey experiment [35] we have  $a_e^0 \lesssim 4 \times 10^{-2}$  for  $\Delta m_{41}^2 \gtrsim 0.1 \text{ eV}^2$ . Now we will consider the effective Majorana mass  $|\langle m \rangle|$  in Schemes A and B.

### A. Scheme A

With the assumption  $m_{1,2} \ll m_{3,4}$  and taking into account the heaviest neutrino masses, we get

$$|\langle m \rangle| \simeq \left| \sum_{j=3,4} U_{ej}^2 \right| m_4, \quad (2.8)$$

where  $m_4 \simeq \sqrt{\Delta m_{\text{LSND}}^2}$ . From Eq.(2.5) it follows that, up to corrections of order  $10^{-2}$ , the relation

$$\sum_{j=3,4} |U_{ej}|^2 \simeq 1 \quad (2.9)$$

is valid. This allows the approximate parameterization

$$U_{e3} \simeq \cos \theta e^{i\alpha_3}, \quad U_{e4} \simeq \sin \theta e^{i\alpha_4} \quad (2.10)$$

and, therefore, [7]

$$|\langle m \rangle| \simeq \sqrt{1 - \sin^2 2\theta \sin^2 \alpha} m_4, \quad (2.11)$$

where  $\alpha = \alpha_4 - \alpha_3$ . Note that  $\alpha$  does not have any effect in neutrinos oscillations, i.e., it is one of the additional phases in the Majorana case.

If CP is conserved in the lepton sector and if we assume a trivial CP phase for the transformation of the electron under CP,<sup>1</sup> we have the CP transformation

$$e(x) \rightarrow -C e^*(x^0, -\vec{x}), \quad \nu_j(x) \rightarrow \eta_j C \nu_j^*(x^0, -\vec{x}), \quad (2.12)$$

where  $C$  is the charge conjugation matrix. Invariance of the Majorana mass term  $\frac{1}{2} m_j \nu_j^T C^{-1} \nu_j$  under this CP transformation leads to the condition

$$\eta_j = i \rho_j = \pm i \quad (2.13)$$

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<sup>1</sup>A CP phase of the electron has no impact on our discussion.

for the phases  $\eta_j$ , which are called CP parities of the Majorana fields  $\nu_j$ . Thus, with Eqs.(2.12) and (2.13) we obtain for Majorana neutrinos

$$U_{ej} = U_{ej}^* \eta_j. \quad (2.14)$$

From Eq.(2.14) it follows that

$$\alpha_j = \frac{\pi}{4} \rho_j + n_j \pi, \quad (2.15)$$

where the  $n_j$  are integers. Therefore, in the case of CP conservation,  $\sin^2 \alpha = 0$  for equal CP parities and  $\sin^2 \alpha = 1$  for opposite CP parities.

From Eq.(2.11) it follows that

$$\sqrt{1 - \sin^2 2\theta} \sqrt{\Delta m_{\text{LSND}}^2} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m_{\text{LSND}}^2}, \quad (2.16)$$

where the boundary values correspond to CP conservation. The angle  $\theta$  can be determined from the analysis of the results of solar neutrino experiments. In fact, the survival probability of solar  $\nu_e$ 's in Scheme A is given by [7]

$$P_{\nu_e \rightarrow \nu_e}^{\text{solar}} = \left(1 - \sum_{j=1,2} |U_{ej}|^2\right)^2 P_{\nu_e \rightarrow \nu_e}(\sin^2 2\theta_{\text{solar}}, \Delta m_{\text{solar}}^2) + \sum_{j=1,2} |U_{ej}|^4. \quad (2.17)$$

Here  $P_{\nu_e \rightarrow \nu_e}(\sin^2 2\theta_{\text{solar}}, \Delta m_{\text{solar}}^2)$  is the standard two-neutrino  $\nu_e$  survival probability with

$$\begin{aligned} \cos^2 \theta_{\text{solar}} &= \frac{|U_{e3}|^2}{1 - \sum_{j=1,2} |U_{ej}|^2} \simeq |U_{e3}|^2, \\ \sin^2 \theta_{\text{solar}} &= \frac{|U_{e4}|^2}{1 - \sum_{j=1,2} |U_{ej}|^2} \simeq |U_{e4}|^2. \end{aligned} \quad (2.18)$$

Comparing Eqs.(2.10) and (2.18) we conclude that

$$\sin^2 2\theta = \sin^2 2\theta_{\text{solar}}. \quad (2.19)$$

It is well known that from the analysis of solar neutrino data two matter MSW solutions and one vacuum oscillation (VO) solution of the solar neutrino problem have been found (see the recent analyses in [36–38] and references therein). To get an idea of the values of  $\sin^2 2\theta_{\text{solar}}$ , we quote the best-fit values of the combined analysis of Ref. [38] for the MSW solutions and for the VO solution the best fit-value of Ref. [36], which takes into account the event rates measured in the solar neutrino experiments:

1. the small mixing angle MSW solution (SMA) with  $\sin^2 2\theta_{\text{solar}} = 4.5 \times 10^{-3}$  and  $\Delta m_{\text{solar}}^2 = 6.3 \times 10^{-6} \text{ eV}^2$  for transitions of solar  $\nu_e$ 's into active neutrinos and  $\sin^2 2\theta_{\text{solar}} = 3.2 \times 10^{-3}$  and  $\Delta m_{\text{solar}}^2 = 5.0 \times 10^{-6} \text{ eV}^2$  for transitions into sterile neutrinos,
2. the large mixing angle MSW solution (LMA) with  $\sin^2 2\theta_{\text{solar}} = 0.80$  and  $\Delta m_{\text{solar}}^2 = 3.6 \times 10^{-5} \text{ eV}^2$ ,

3. the vacuum oscillation solution (VO) with  $\sin^2 2\theta_{\text{solar}} = 0.75$  and  $\Delta m_{\text{solar}}^2 = 8.0 \times 10^{-11} \text{ eV}^2$ .

Note that for transitions into sterile neutrinos the data allow only the SMA MSW solution. From Eq.(2.16) for the SMA MSW solution we get the relation

$$|\langle m \rangle| \simeq \sqrt{\Delta m_{\text{LSND}}^2}, \quad (2.20)$$

which is independent of CP violation. If this possibility is realized in nature,  $(\beta\beta)_{0\nu}$  experiments are expected to see an effect in the near future if  $\sqrt{\Delta m_{\text{LSND}}^2} \gtrsim 0.5 \text{ eV}$ . If, however, it will be found that  $|\langle m \rangle| \ll 0.5 \text{ eV}$ , within Scheme A with Majorana neutrinos the SMA MSW solution of the solar neutrino problem will be incompatible with the data. Thus, an experimental confirmation of the SMA MSW solution of the solar neutrino problem and the indicated stringent upper limit on  $|\langle m \rangle|$  would allow to rule out Scheme A with massive Majorana neutrinos.

It was shown in Ref. [34] that, if the effective number  $N_\nu$  of neutrinos relevant in big bang nucleosynthesis is smaller than 4, then in both Schemes A and B the solution to the solar neutrino puzzle in the framework of neutrino oscillations is given by transitions into sterile neutrinos and consequently by the SMA MSW solution. If  $N_\nu < 4$  is not correct,  $\nu_e$  could in principle make transitions into an arbitrary admixture of  $\nu_\tau$  and  $\nu_s$  in Schemes A and B. However, for the LMA MSW and VO solutions an analysis of the solar neutrino data should certainly put a constraint on the admixture of sterile neutrinos.

With increasing accuracy of the Super-Kamiokande measurements of the day-night asymmetry and the electron recoil energy spectrum and with future results of the SNO experiment [39] the preferred solution of the solar neutrino problem can possibly be found [28]. In the case of the LMA MSW or the VO solution, assuming  $|\langle m \rangle|$  and  $\sin^2 2\theta_{\text{solar}}$  to be measured, we arrive at

$$\sin^2 \alpha \simeq \frac{1}{\sin^2 2\theta_{\text{solar}}} \left( 1 - \frac{|\langle m \rangle|^2}{\Delta m_{\text{LSND}}^2} \right) \quad (2.21)$$

from Eq.(2.11). Thus, if future measurements show the correctness of a large mixing angle solution of the solar neutrino problem (VO or MSW), then from measurements of  $|\langle m \rangle|$  information on the Majorana phase  $\alpha$  of the mixing matrix  $U$  can be obtained.

Finally, in Scheme A the neutrino mass which is probed in  $^3\text{H}$   $\beta$ -decay spectrum is given by

$$m(^3\text{H}) \simeq m_4 \simeq \sqrt{\Delta m_{\text{LSND}}^2}. \quad (2.22)$$

For  $\sqrt{\Delta m_{\text{LSND}}^2} \sim 1 \text{ eV}$ ,  $m(^3\text{H})$  lies in the sensitivity region of future  $^3\text{H}$  experiments [40]. The check of the relation (2.22) is an additional test of Scheme A.

## B. Scheme B

In Scheme B we have



$$\sum_{j=3,4} |U_{ej}|^2 \leq a_e^0. \quad (2.23)$$

Thus the contribution of the heavy masses  $m_3 \simeq m_4 \simeq \sqrt{\Delta m_{\text{LSND}}^2}$  to the effective Majorana mass  $|\langle m \rangle|$  is suppressed in this scheme. Taking into account also the light masses  $m_{1,2}$ , we obtain the bound

$$|\langle m \rangle| \lesssim |\langle m \rangle|_2 + |\langle m \rangle|_{34}, \quad (2.24)$$

where

$$|\langle m \rangle|_{34} = a_e^0 \sqrt{\Delta m_{\text{LSND}}^2} \quad (2.25)$$

is the upper bound of the contribution of  $\nu_{3,4}$  to  $|\langle m \rangle|$  and, with the assumption  $m_1 \ll \sqrt{\Delta m_{\text{solar}}^2}$ ,

$$|\langle m \rangle|_2 = \sin^2 \theta_{\text{solar}} \sqrt{\Delta m_{\text{solar}}^2} \quad (2.26)$$

is the contribution of  $\nu_2$  with

$$\sin^2 \theta_{\text{solar}} = \frac{1}{2} \left( 1 - \sqrt{1 - \sin^2 2\theta_{\text{solar}}} \right). \quad (2.27)$$

Note that in the above consideration we have neglected  $m_1$ . If  $m_1$  and  $m_2$  are of the same order of magnitude, we have

$$|\langle m \rangle|_2 = m_1 + \sin^2 \theta_{\text{solar}} \frac{\Delta m_{\text{solar}}^2}{m_1 + \sqrt{m_1^2 + \Delta m_{\text{solar}}^2}}, \quad (2.28)$$

which reduces to Eq.(2.26) for  $m_1 \ll \sin^2 \theta_{\text{solar}} \sqrt{\Delta m_{\text{solar}}^2}$ . As long as  $m_1 \lesssim \sqrt{\Delta m_{\text{solar}}^2}$  holds, the numerical value of the second term of the bound  $|\langle m \rangle|_2$  does not change very much as a function of  $m_1$ .

### III. NEUTRINOLESS DOUBLE $\beta$ -DECAY IN THREE-NEUTRINO SCHEMES

If the LSND indications in favour of  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  oscillations are not confirmed by future experiments it is sufficient to assume the existence of only three light massive neutrinos. With  $m_1 \ll m_3$  there are two possible mass spectra in this case:

- I. *The hierarchical spectrum* defined by<sup>2</sup>  $m_1 \ll m_2 \ll m_3$  with  $\Delta m_{21}^2 = \Delta m_{\text{solar}}^2$  and  $\Delta m_{31}^2 = \Delta m_{\text{atm}}^2$ .
- II. *The spectrum with reversed hierarchy* defined by  $m_3 > m_2 \gg m_1$  with  $\Delta m_{32}^2 = \Delta m_{\text{solar}}^2$  and  $\Delta m_{31}^2 = \Delta m_{\text{atm}}^2$ . In this case  $m_2$  and  $m_3$  are quasi-degenerate.

For a discussion of  $|\langle m \rangle|$  with large  $m_1$ , allowing thus for degeneracy of the three neutrino masses, see Refs. [5,13,14].

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<sup>2</sup>For the calculation of  $|\langle m \rangle|$  we will also allow for  $m_1 \sim m_2$ .

## A. Neutrino mass hierarchy

In the case of the hierarchical neutrino mass spectrum (see also footnote 2) we have the upper bound

$$|\langle m \rangle| \lesssim |\langle m \rangle|_2 + |\langle m \rangle|_3, \quad (3.1)$$

where

$$|\langle m \rangle|_3 = |U_{e3}|^2 \sqrt{\Delta m_{\text{atm}}^2} \quad (3.2)$$

and  $|\langle m \rangle|_2$  is given by Eq.(2.28).<sup>3</sup>

From the result of the LBL reactor experiment CHOOZ [31], if one takes into account that  $|U_{e3}|^2$  cannot be large because of the results of the solar neutrino experiments, one can find an upper bound on  $|U_{e3}|^2$ . With the upper bound  $B_{e;e}^{\text{CHOOZ}}$  on the amplitude of  $\bar{\nu}_e \rightarrow \bar{\nu}_e$  oscillations we have

$$|U_{e3}|^2 \leq \frac{1}{2} \left( 1 - \sqrt{1 - B_{e;e}^{\text{CHOOZ}}} \right), \quad (3.3)$$

and from the 90% CL CHOOZ exclusion plot one obtains  $|U_{e3}|^2 \lesssim 5 \times 10^{-2}$  for  $\Delta m_{31}^2 = \Delta m_{\text{atm}}^2 \gtrsim 2 \times 10^{-3} \text{ eV}^2$ . From a 3-neutrino analysis of the Super-Kamiokande + CHOOZ data, in Ref. [41] it was found that  $|U_{e3}|^2 \lesssim 0.15$  at 90% CL, valid in the whole range of  $\Delta m_{\text{atm}}^2$ . A similar result has been derived in Ref. [42] ( $|U_{e3}|^2 \lesssim 0.1$  at 95% CL). These bounds on  $|U_{e3}|^2$  allow us to evaluate  $|\langle m \rangle|_3$  (3.2) (see summary).

## B. The reversed hierarchy

If this neutrino mass spectrum is realized in nature then the bounds on  $|U_{e3}|^2$  mentioned before are now valid for  $|U_{e1}|^2$ . Thus the contribution of  $m_1$  to  $|\langle m \rangle|$  (1.4) can be neglected and we obtain

$$|\langle m \rangle| \simeq \left| \sum_{j=2,3} U_{ej}^2 \right| \sqrt{\Delta m_{31}^2} \quad (3.4)$$

with  $\Delta m_{31}^2 = \Delta m_{\text{atm}}^2$ . Taking into account that  $\sum_{j=2,3} |U_{ej}|^2$  is close to 1 and that  $\Delta m_{32}^2 = \Delta m_{\text{solar}}^2$ , then with the methods of Subsection II A we arrive at the expression [7]

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<sup>3</sup>If we include the effect of non-zero  $U_{e3}$ , then we have

$$|\langle m \rangle|_2 = (1 - |U_{e3}|^2) \left( m_1 + \sin^2 \theta_{\text{solar}} \frac{\Delta m_{\text{solar}}^2}{m_1 + \sqrt{m_1^2 + \Delta m_{\text{solar}}^2}} \right).$$

$$|\langle m \rangle| \simeq \sqrt{1 - \sin^2 2\theta_{\text{solar}} \sin^2 \alpha} \sqrt{\Delta m_{\text{atm}}^2}, \quad (3.5)$$

where now  $\alpha = \alpha_3 - \alpha_2$  and  $\alpha_{2,3}$  are the phases of  $U_{e2}$  and  $U_{e3}$ , respectively. The discussion in Subsection II A on the possibility of the determination of the CP phase  $\alpha$  is also applicable here (for further details see [7]). From Eq.(3.5) we get the bounds

$$\sqrt{1 - \sin^2 2\theta_{\text{solar}}} \sqrt{\Delta m_{\text{atm}}^2} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m_{\text{atm}}^2}, \quad (3.6)$$

analogous to Eq.(2.16) with  $\Delta m_{\text{LSND}}^2$  replaced by  $\Delta m_{\text{atm}}^2$ .

#### IV. SUMMARY

Now we want to summarize our discussion of the effective Majorana mass  $|\langle m \rangle|$  relevant in  $(\beta\beta)_{0\nu}$  decay and present some numerical estimates. Using input from existing data on neutrino oscillations and under the assumption of small  $m_1$ , the following approximate bounds on (or values of)  $|\langle m \rangle|$  are obtained. The concrete results depend on the nature of the solution of the solar neutrino problem.

##### ◇ 4-neutrino schemes

###### □ Scheme A:

$$\sqrt{1 - \sin^2 2\theta_{\text{solar}}} \sqrt{\Delta m_{\text{LSND}}^2} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m_{\text{LSND}}^2}$$

If the SMA solution of the solar neutrino problem is the correct one, then  $|\langle m \rangle| \simeq \sqrt{\Delta m_{\text{LSND}}^2}$  with

$$0.5 \text{ eV} \lesssim \sqrt{\Delta m_{\text{LSND}}^2} \lesssim 1.4 \text{ eV}$$

at 90% CL [29]. Concerning the LMA solutions, the lower bound on  $|\langle m \rangle|$  depends strongly on the upper bound on  $\sin^2 2\theta_{\text{solar}}$ . In Ref. [38] it was found for the LMA MSW solution in the combined analysis of all solar neutrino data that  $\sin^2 2\theta_{\text{solar}} \leq 0.97$  at 90% CL (also at 99% CL)<sup>4</sup>. Using this value, we have plotted in Fig. 1 the region defined by the above inequalities. The shaded region represents the possible values of  $|\langle m \rangle|$  in the allowed range of  $\Delta m_{\text{LSND}}^2$ . We can read off from Fig. 1 that the effective Majorana mass lies in the range  $7 \times 10^{-2} \lesssim |\langle m \rangle| \lesssim 1.4 \text{ eV}$ . Note that for the VO solution  $\sin^2 2\theta_{\text{solar}}$  could be as large as 1 and the lower bound in Fig. 1 disappears in this case.

###### □ Scheme B:

$$|\langle m \rangle| \lesssim a_e^0 \sqrt{\Delta m_{\text{LSND}}^2} + m_1 + \sin^2 \theta_{\text{solar}} \frac{\Delta m_{\text{solar}}^2}{m_1 + \sqrt{m_1^2 + \Delta m_{\text{solar}}^2}}$$

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<sup>4</sup>We are indebted to M.C. Gonzalez-Garcia for providing us with this number.

For the SMA MSW solution and the VO solution this inequality reduces to  $|\langle m \rangle| \lesssim a_e^0 \sqrt{\Delta m_{\text{LSND}}^2} + m_1$ . With  $a_e^0$  (2.7) as a function of  $\Delta m_{\text{LSND}}^2$  one obtains

$$a_e^0 \sqrt{\Delta m_{\text{LSND}}^2} \lesssim 2 \times 10^{-2} \text{ eV}$$

in the allowed range of  $\Delta m_{\text{LSND}}^2$  [12]. In the case of the LMA MSW solution also the third term in the above inequality contributes. Using the results of Ref. [38] one gets approximately

$$\sin^2 \theta_{\text{solar}} \sqrt{\Delta m_{\text{solar}}^2} \lesssim 0.3 \times 10^{-2} \text{ eV} \quad (\text{LMA MSW})$$

from the 90% CL plot of the combined analysis. For  $m_1 \sim \sqrt{\Delta m_{\text{solar}}^2}$  the quantity  $|\langle m \rangle|_2$  is, therefore, of the order of  $10^{-2}$  eV.

### ▽ 3-neutrino schemes

#### △ Neutrino mass hierarchy:

$$|\langle m \rangle| \lesssim |U_{e3}|^2 \sqrt{\Delta m_{\text{atm}}^2} + m_1 + \sin^2 \theta_{\text{solar}} \frac{\Delta m_{\text{solar}}^2}{m_1 + \sqrt{m_1^2 + \Delta m_{\text{solar}}^2}}$$

The term with  $\sin^2 \theta_{\text{solar}}$  is estimated as for Scheme B. From the consideration in Subsection III A it is clear that the first term in this inequality cannot be larger than about  $10^{-2}$ . A numerical evaluation of this term shows that [12,17]

$$|U_{e3}|^2 \sqrt{\Delta m_{\text{atm}}^2} \lesssim 0.6 \times 10^{-2} \text{ eV}.$$

Thus, with  $m_1 = 0$  we have  $|\langle m \rangle| \lesssim 0.6 \times 10^{-2}$  eV in the case of the VO or SMA MSW solution, while for the LMA MSW solution one finds  $|\langle m \rangle| \lesssim 0.9 \times 10^{-2}$  eV.

#### △ Reversed hierarchy:

$$\sqrt{1 - \sin^2 2\theta_{\text{solar}}} \sqrt{\Delta m_{\text{atm}}^2} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m_{\text{atm}}^2}$$

For the SMA MSW solution we have  $|\langle m \rangle| \simeq \sqrt{\Delta m_{\text{atm}}^2}$  with the range of  $\sqrt{\Delta m_{\text{atm}}^2}$  given approximately by [1]

$$0.03 \text{ eV} \lesssim \sqrt{\Delta m_{\text{atm}}^2} \lesssim 0.1 \text{ eV}.$$

For the LMA solutions the remarks for Scheme A are valid. The possible values of  $|\langle m \rangle|$  in the case of the LMA MSW solution are shown by the shaded area in Fig. 2, from where we get the range  $6 \times 10^{-3} \lesssim |\langle m \rangle| \lesssim 0.1$  eV.

In conclusion, assuming  $m_1 \ll m_4$  in the four-neutrino scenarios, it is possible to have  $|\langle m \rangle|$  as large as  $\sim 1$  eV in Scheme A. In the three-neutrino scheme with two quasi-degenerate neutrinos and reversed hierarchy ( $m_1 \ll m_2 \simeq m_3$ ),  $|\langle m \rangle|$  could be as large as  $\sim 0.1$  eV. However, in four-neutrino Scheme B and in the hierarchical three-neutrino scheme, the

effective Majorana mass  $|\langle m \rangle|$  is strongly suppressed, with bounds of order  $10^{-2}$  eV. If in future  $(\beta\beta)_{0\nu}$  experiments it is found that  $|\langle m \rangle| \gg 10^{-2}$  eV it would mean that Scheme B and the mass hierarchy with three neutrinos are excluded, or that  $(\beta\beta)_{0\nu}$  decay proceeds via other mechanisms [23], not involving the effective Majorana mass (1.4). We would like to remind the reader that, in addition to Scheme A, one can have  $|\langle m \rangle| > 0.1$  eV also in a three neutrino mixing scheme with three quasi-degenerate neutrinos [5,13,14].

The results presented here demonstrate that  $(\beta\beta)_{0\nu}$  decay experiments are not only important in the context of revealing the Dirac or Majorana nature of neutrinos but also for the determination of the character of the neutrino mass spectrum.

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# FIGURES

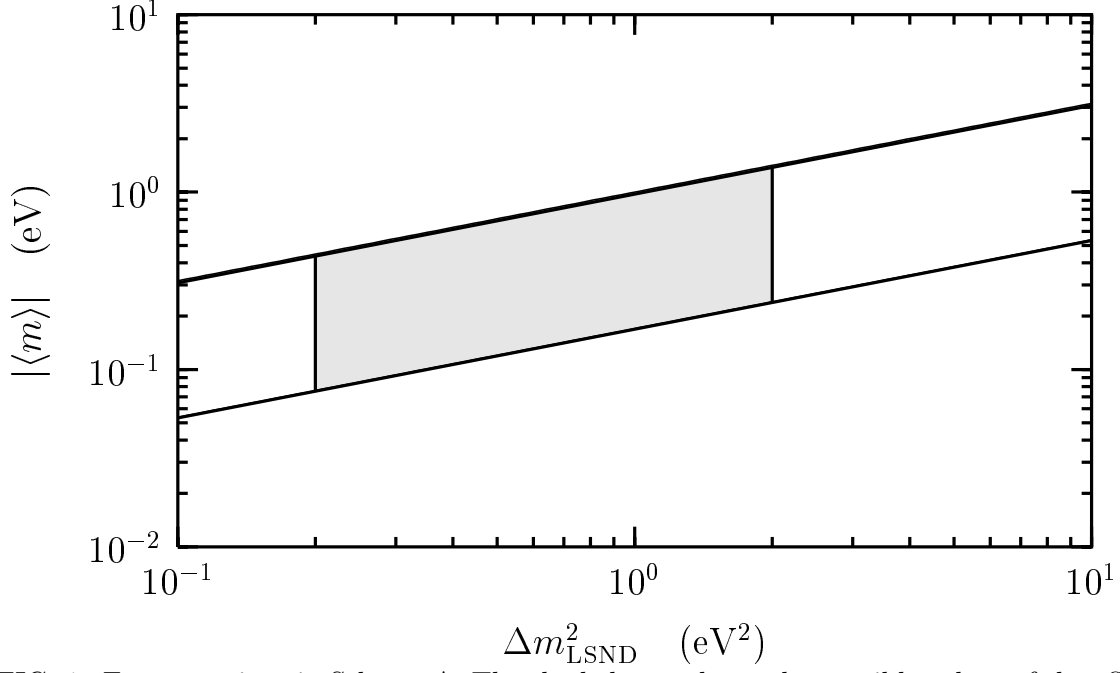


FIG. 1. Four neutrinos in Scheme A: The shaded area shows the possible values of the effective Majorana mass  $|\langle m \rangle|$  in the range of  $\Delta m_{\text{LSND}}^2$  for the case of the LMA MSW solution of the solar neutrino problem.

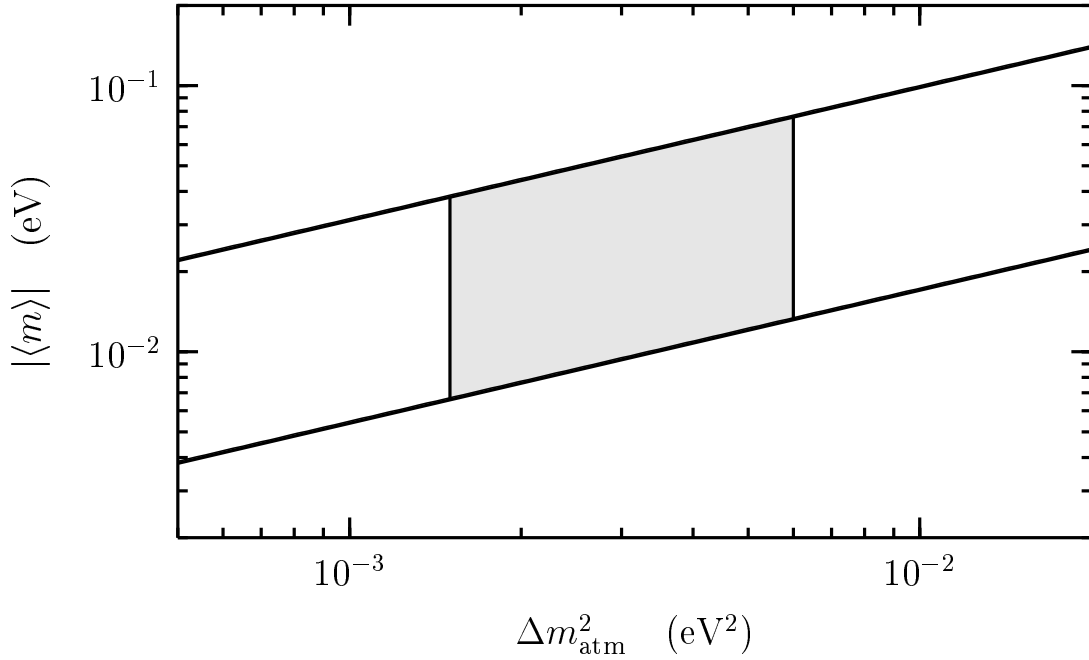


FIG. 2. Three neutrinos with reversed mass hierarchy: The shaded area shows the possible values of the effective Majorana mass  $|\langle m \rangle|$  in the range of  $\Delta m_{\text{atm}}^2$  for the case of the LMA MSW solution of the solar neutrino problem.